Components of Decision Theory

- **Decision Maker-**

A person who takes decision is called a decision maker. He may be a shopkeeper , manufacturer , businessman

- **Course of Action –**

These are different ways , acts or options available to the decision maker.

These could be different levels of production eg 300 units , 500 units , 800 units

Different type of production eg. Type I , Type II , Type III

Vender from whom the raw material to be purchased eg A , B , C

They are also known as Acts and are denoted by A_1, A_2---- An

- **States of nature**

They represent all future event that can occur. They may be natural possibilities , market situations or even government policies. States of nature are also known as Events and are denoted as S_1, S_2, S_3------- Sn.

Decision maker has no control on Events.

- **Pay off –**

Each combination of State of nature & Course of action is associated with an amount. This amount is pay-off for that particular combination of Act and event. It could be profit – represented by positive pay-off or

loss represented by negative pay-off

	S1	S2	----------	Sn
A1				
A2				
:				
An				

- Decision making environment can be broadly classified into

➢ Under Certainty

➢ Under Uncertainty

➢ Under Risk

➢ Under Conflict

We will see Decision making Under Uncertainty & Under Risk

Decision Making Under Uncertainty

- **<u>Maximin Criterion</u>**

Steps

1. Choose minimum payoff for each Act
2. Select maximum out of these minimum.
3. Corresponding Act is a best decision

This criterion is also known as criteria of pessimism because it reflects pessimistic attitude of the decision maker.

- **Maximax Criterion**

Steps

1. Choose maximum payoff for each Act
2. Select maximum out of these minimum.
3. Corresponding Act is a best decision

This criterion is also known as criteria of optimism because it reflects optimistic attitude of the decision maker.

- **Laplace Criterion**

Steps

1. Calculate average pay off for each Act
2. Choose maximum out of these averages
3. Corresponding Act is a best decision

- **Minimax Regret Criterion**

Steps

1. Choose maximum payoff for each Event
2. Calculate maximum payoff – individual payoff for each combination of Act &Event. This is known as Regret value for that particular combination of Act and Event
3. Select maximum Regret for each course of Action.
4. Select minimum out of these maximum.
5. Corresponding Act is a best decision

Q1. Given the following payoff matrix , select the best decision by

i. Maximax

ii. Maximin

iii. Laplace

iv. Minimax Regret Criterion

	S1	S2	S3
A1	2500	2500	2500
A2	4000	3500	2000
A3	2500	2000	1200

Answer:

	S1	S2	S3	Min	Max	Avg
A1	2500	2500	2500	2500	2500	2500
A2	4000	3500	2000	2000	4000	3166.67
A3	2500	2000	1200	1200	2500	1900

MAXIMAX = MAX (2500 , 4000 , 2500) = 4000

A2 is the best decision

MAXIMIN = MAX (2500 , 2000 ,1200) = 2500

A1 is the best decision

LAPLACE = MAX (2500 , 3166.67 , 1900) = 3166.67

A2 is the best decision

MINIIMAX = MIN (1500 , 500 , 1500) = 500

A2 is the best decision

MINIMAX REGRET CRITERIA

	S1	S2	S3
A1	2500	2500	2500
A2	4000	3500	2000
A3	2500	2000	1200

REGRET TABLE

	S1	S2	S3	Max
A1	1500	1000	0	1500
A2	0	0	500	500
A3	1500	1500	1300	1500

Q2. Find the best decision using

i. Maximax
ii. Maximin
iii. Laplace
iv. Minimax Regret Criterion

	S1	S2	S3
	S1	S2	S3
A1	100	150	190
A2	350	200	0
A3	-50	160	400

	S1	S2	S3	Min	Max	Average
A1	100	150	190	100	190	146.66
A2	350	200	0	0	350	183.33
A3	-50	160	400	-50	400	170

Answer :

i. Maximax Criterion = Max (190 , 350 , 400) = 400
A3 is the best decision

ii. Maximin Criterion = Max (100 , 0 , -50) = 100
A1 is the best decision

iii. Laplace Criterion = Max (146.66 , 183.33 , 170) = 183.33
A2 is the best decision

iv. Minimax Regret Criterion

	S_1	S_2	S_3
A_1	100	150	190
A_2	350	200	0
A_3	-50	160	400

Regret Table

	S_1	S_2	S_3	Max
A_1	250	50	210	250
A_2	0	0	400	400
A_3	400	40	0	400

Minimax Regret Criterion = Min (250 ,400 ,400) = 250

A_1 is the best decision

Q3. Find the best decision using

i. Maximax
ii. Maximin
iii. Laplace
iv. Minimax Regret Criterion

	S_1	S_2	S_3	S_4
A_1	2500	4000	-1800	3000
A_2	4500	-1000	0	5000
A_3	6000	8500	12000	10000

Answer:

	S_1	S_2	S_3	S_4	Min	Max	Avg
A_1	2500	4000	-1800	3000	-1800	4000	1925
A_2	4500	-1000	0	5000	-1000	5000	2125
A_3	6000	8500	12000	10000	6000	12000	9125

i. Maximax Criterion = Max (4000 , 5000 , 12000) = 12000
A3 is the best decision

ii. Maximin Criterion = Max (-1800 , -1000 , 6000) = 6000
A3 is the best decision

iii. Laplace Criterion = Max (1925 , 2125 , 9125) = 9125

A3 is the best decision

iv. Minimax Regret Criterion

	S1	S2	S3	S4
A1	2500	4000	-1800	3000
A2	4500	-1000	0	5000
A3	6000	8500	12000	10000

Regret Table

	S1	S2	S3	S4	Max
A1	3500	4500	13800	7000	13800
A2	1500	9500	12000	5000	12000
A3	0	0	0	0	0

Minimax Regret Criterion = Min (13800 ,12000 ,0) = 0
A3 is the best decision

Decision Making Under Risk

Here the decision maker has knowledge of probabilities of occurrence of each State of nature.

Steps

EMV (Expected Monetary Value) Criterion

1. EMV is calculated for each course of action by taking sum of products of pay-offs & respective probabilities
2. Select the Maximum EMV
3. Corresponding Act is the optimum decision.

EOL (Expected Opportunity Loss) Criterion

1. Calculate Regret Table
2. EOL is calculated for each course of action by taking sum of products of Regret Values & respective probabilities
3. Select the Minimum EOL
4. Corresponding Act is the optimum decision.

Q1. Obtain the best decision using EMV & EOL Criterion

	S_1	S_2	S_3
A_1	2500	2500	2500
A_2	4000	3500	2000
A_3	5000	2500	1000
Probability	0.4	0.5	0.1

$EMV(A_1) = 2500 \times 0.4 + 2500 \times 0.5 + 2500 \times 0.1 = 2500$

$EMV(A_2) = 4000 \times 0.4 + 3500 \times 0.5 + 2000 \times 0.1 = 3550$

EMV(A3) = 5000 x 0.4 + 2500 x 0.5 + 1000 x 0.1 = 3350

EMV is Maximum for A2
A2 is the best decision. Maximum EMV = 3550

Calculation of EOL

	S1	S2	S3
A1	2500	2500	2500
A2	4000	3500	2000
A3	5000	2500	1000
Probability	0.4	0.5	0.1

Regret Table

	S1	S2	S3
A1	2500	1000	0
A2	1000	0	500
A3	0	1000	1500
Probability	0.4	0.5	0.1

EOL(A1) = 2500 x 0.4 + 1000 x 0.5 + 0 x 0.1 = 1500
EOL(A2) = 1000 x 0.4 + 0 x 0.5 + 500 x 0.1 = 450
EOL(A3) = 0 x 0.4 + 1000 x 0.5 +1500 x 0.1 = 650

EOL is Minimum for A2
A2 is the best decision. Minimum EOL = 450

Q2. Obtain the best decision using EMV & EOL Criterion

	S1	S2	S3
A1	-80	60	110
A2	40	0	-50
A3	100	-20	70
Probability	0.3	0.2	0.5

EMV(A1) = -80 x 0.3 + 60 x 0.2 + 110 x 0.5

= -24 +12+ 55 = 43

EMV(A2) = 40 x 0.3 + 0 x 0.2 - 50 x 0.5

= 12 +0- 25 = -13

EMV(A3) = 100 x 0.3 - 20 x 0.2 + 70 x 0.5

= 30 -4+ 35 = 61

EMV is Maximum for A3

A3 is the best decision. Maximum EMV = 61

Calculation of EOL

	S1	S2	S3
A1	-80	60	110
A2	40	0	-50
A3	100	-20	70
Probability	0.3	0.2	0.5

Regret Table

	S1	S2	S3
A1	180	0	0
A2	60	60	160
A3	0	80	40
Probability	0.3	0.2	0.5

EOL(A1) = 180 x 0.3 + 0 x 0.2 + 0 x 0.5 = 54

EOL(A2) = 60 x 0.3 + 60 x 0.2 + 160 x 0.5 = 18 + 12 +80 = 110

EOL(A3) = 0 x 0.3 +80 x 0.2 + 40 x 0.5 = 0+ 16 + 20 = 36

EOL is Minimum for A3

A3 is the best decision. Minimum EOL = 36

Q3. Obtain the best decision using EMV & EOL Criterion

	S1	S2	S3	S4
A1	100	600	-300	100
A2	800	0	200	0
A3	-100	400	0	200
A4	0	600	600	0
Probability	0.15	0.45	0.25	0.15

EMV(A1) = 100 x 0.15 + 600 x 0.45 - 300 x 0.25 + 100 x 0.15
= 15 +270 -75 + 15= 225
EMV(A2) = 800 x 0.15 + 0 x 0.45 + 200 x 0.25 + 0 x 0.15
= 120 +0 +50 + 0= 170
EMV(A3) = -100 x 0.15 + 400 x 0.45 + 0 x 0.25 + 200 x 0.15
= -15 +180 +0 + 30= 195
EMV(A4) = 0 x 0.15 + 600 x 0.45 + 600 x 0.25 + 0 x 0.15
= 0+270 +150 +0= 420

EMV is Maximum for A4
A4 is the best decision. Maximum EMV = 420

Calculation of EOL

	S1	S2	S3	S4
A1	100	600	-300	100
A2	800	0	200	0
A3	-100	400	0	200
A4	0	600	600	0
Probability	0.15	0.45	0.25	0.15

Regret Table

	S_1	S_2	S_3	S_4
A_1	700	0	900	100
A_2	0	600	400	200
A_3	900	200	600	0
A_4	800	0	0	200
Probability	0.15	0.45	0.25	0.15

$EOL(A_1) = 700 \times 0.15 + 0 \times 0.45 + 900 \times 0.25 + 100 \times 0.15$

$= 105 + 0 + 225 + 15 = 345$

$EOL(A_2) = 0 \times 0.15 + 600 \times 0.45 + 400 \times 0.25 + 200 \times 0.15$

$= 0 + 270 + 100 + 30 = 400$

$EOL(A_3) = 900 \times 0.15 + 200 \times 0.45 + 600 \times 0.25 + 0 \times 0.15$

$= 135 + 90 + 150 + 0 = 375$

$EOL(A_4) = 800 \times 0.15 + 0 \times 0.45 + 0 \times 0.25 + 200 \times 0.15$

$= 120 + 0 + 0 + 30 = 150$

EOL is Minimum for A_4

A_4 is the best decision. Minimum EOL = 150

Decision Tree

Decision tree is a graphical representation of a decision process involving multiple stages. It indicates all the courses of actions , States of nature with associated probabilities and conditional payoffs of a decision problem.

A decision node is represented by a square and states of nature node is represented by a circle. Usually decision nodes are numbered using A,B,C,.. from left to right and states of nature nodes are numbered as 1,2,3… sequentially from left to right.

Different courses of action originate as branches from the decision node. At the end of each branches , there are states of nature nodes. These states of nature emerge as sub branches from these nodes. The associated probabilities and pay offs are shown on the branches.

Expected Monetary Value are calculated for each course of action. The Course of action with maximum EMV is the optimum decision.

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